

# Financial Sensitivity

## Financial Sensitivity Definition

- ◆ Financial sensitivity is the measure of the value reaction of a financial instrument to changes in underlying factors.
- ◆ The value of a financial instrument is impacted by many factors, such as interest rate, stock price, implied volatility, time, etc.
- ◆ Financial sensitivities are also called Greeks, such as Delta, Gamma, Vega and Theta.
- ◆ Financial sensitivities are risk measures that are more important than fair values.
- ◆ They are vital for risk management: isolating risk, hedging risk, explaining profit and loss, etc.

## Delta Definition

- ◆ Delta is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying asset price.

- ◆ Interest rate Delta:

$$IrDelta = \frac{\partial V}{\partial r} = \frac{V(r + 0.0001) - V(r)}{0.0001}$$

where  $V(r)$  is the instrument value and  $r$  is the underlying interest rate.

- ◆ PV01, or dollar duration, is analogous to interest rate Delta but has the change value of a one-dollar annuity given by

$$PV01 = V(r + 0.0001) - V(r)$$

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## Vega Definition

- ◆ Vega is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying implied volatility.

$$Vega = \frac{\partial V}{\partial \sigma} = \frac{V(\sigma + \Delta\sigma) - V(\sigma)}{\Delta\sigma}$$

where  $\sigma$  is the implied volatility.

- ◆ Only non-linear products, such as options, have Vegas.

## Gamma Definition

- ◆ Gamma is a second order Greek that measures the value change of a financial instrument with respect to changes in the underlying price.

$$Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{V(S + 0.5 * \Delta S) + V(S - 0.5 * \Delta S) - 2V(S)}{\Delta S^2}$$

## Theta Definition

- ◆ Theta is a first order Greek that measures the value change of a financial instrument with respect to time.

$$Theta = \frac{\partial V}{\partial t} = \frac{V(t + \Delta t) - V(t)}{\Delta t}$$

## Curvature Definition

- ◆ Curvature is a new risk measure for options introduced by Basel FRTB.
- ◆ It is a risk measure that captures the incremental risk not captured by the delta risk of price changes in the value of an option.

$$Curvature = \min\{V(S + \Delta W) - V(S) - \Delta W * Delta, V(S - \Delta W) - V(S) - \Delta W * Delta\}$$

where  $\Delta W$  is the risk weight.

## Sensitivity Hedging

- ◆ The objective of hedging is to have a lower price volatility that eliminates both downside risk (loss) and upside profit.
- ◆ Hedging is a double-edged sword.
- ◆ The profit of a broker or an investment bank comes from spread rather than market movement. Thus it is better to hedge all risks.
- ◆ Delta is normally hedged.
- ◆ Vega can be hedged by using options.
- ◆ Gamma is hardly hedged in real world.

## Sensitivity Profit & Loss (P&L)

- ◆ Hypothetic P&L is the P&L that is purely driven by market movement.
- ◆ Hypothetic P&L is calculated by revaluing a position held at the end of the previous day using the market data at the end of the current day, i.e.,

$$\text{Hypothetical P\&L} = V(t-1, P_{t-1}, M_t) - V(t-1, P_{t-1}, M_{t-1})$$

where  $t-1$  is yesterday;  $t$  is today;  $P_{t-1}$  is the position at yesterday;  $M_{t-1}$  is yesterday's market and  $M_t$  is today's market.

- ◆ Sensitivity P&L is the sum of Delta P&L, Vega P&L and Gamma P&L.
- ◆ Unexplained P&L = Hypothetical P&L - Sensitivity P&L.

## Backbone Adjustment

- ◆ Backbone adjustment is an advanced topic in sensitivity P&L.
- ◆ It can be best explained mathematically.
- ◆ Assume the value of an option is a function of the underlying price  $S$  and implied volatility  $\sigma$ , i.e.,  $V = F(S, \sigma)$ .
- ◆ If the implied volatility is a function of the ATM volatility and strike (sticky strike assumption), i.e.,  $\sigma = \sigma_A + f(K)$ , the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A = \text{DeltaP\&L} + \text{VegaP\&L}$$

$$\text{where } \text{DeltaP\&L} = \frac{\partial F}{\partial S} dS \text{ and } \text{VegaP\&L} = \frac{\partial F}{\partial \sigma_A} d\sigma_A$$



Reference:

<https://finpricing.com/lib/EqConvertible.html>