Financial Sensitivity
Financial Sensitivity Definition

- Financial sensitivity is the measure of the value reaction of a financial instrument to changes in underlying factors.
- The value of a financial instrument is impacted by many factors, such as interest rate, stock price, implied volatility, time, etc.
- Financial sensitivities are also called Greeks, such as Delta, Gamma, Vega and Theta.
- Financial sensitivities are risk measures that are more important than fair values.
- They are vital for risk management: isolating risk, hedging risk, explaining profit and loss, etc.
Delta Definition

- Delta is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying asset price.
- Interest rate Delta:
  \[ \text{IrDelta} = \frac{\partial V}{\partial r} = \frac{V(r + 0.0001) - V(r)}{0.0001} \]
  where \( V(r) \) is the instrument value and \( r \) is the underlying interest rate.
- PV01, or dollar duration, is analogous to interest rate Delta but has the change value of a one-dollar annuity given by
  \[ PV01 = V(r + 0.0001) - V(r) \]
Vega Definition

- Vega is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying implied volatility.

\[
Vega = \frac{\partial V}{\partial \sigma} = \frac{V(\sigma + \Delta \sigma) - V(\sigma)}{\Delta \sigma}
\]

where \( \sigma \) is the implied volatility.

- Only non-linear products, such as options, have Vegas.

Gamma Definition

- Gamma is a second order Greek that measures the value change of a financial instrument with respect to changes in the underlying price.

\[
Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{V(S + 0.5 \cdot \Delta S) + V(S - 0.5 \cdot \Delta S) - 2V(S)}{\Delta S^2}
\]
Theta Definition

- Theta is a first order Greek that measures the value change of a financial instrument with respect to time.

\[
\text{Theta} = \frac{\partial V}{\partial t} = \frac{V(t + \Delta t) - V(t)}{\Delta t}
\]

Curvature Definition

- Curvature is a new risk measure for options introduced by Basel FRTB.
- It is a risk measure that captures the incremental risk not captured by the delta risk of price changes in the value of an option.

\[
\text{Curvature} = \min\{V(S + \Delta W) - V(S) - \Delta W \times \text{Delta}, V(S - \Delta W) - V(S) - \Delta W \times \text{Delta}\}
\]

where \(\Delta W\) is the risk weight.
The objective of hedging is to have a lower price volatility that eliminates both downside risk (loss) and upside profit.

Hedging is a double-edged sword.

The profit of a broker or an investment bank comes from spread rather than market movement. Thus it is better to hedge all risks.

Delta is normally hedged.

Vega can be hedged by using options.

Gamma is hardly hedged in real world.
Sensitivity Profit & Loss (P&L)

◆ Hypothetic P&L is the P&L that is purely driven by market movement.
◆ Hypothetic P&L is calculated by revaluing a position held at the end of the previous day using the market data at the end of the current day, i.e.,

\[
Hypothetical\, P&L = V(t - 1, P_{t-1}, M_t) - V(t - 1, P_{t-1}, M_{t-1})
\]

where \(t - 1\) is yesterday; \(t\) is today; \(P_{t-1}\) is the position at yesterday; \(M_{t-1}\) is yesterday’s market and \(M_t\) is today’s market.

◆ Sensitivity P&L is the sum of Delta P&L, Vega P&L and Gamma P&L.
◆ Unexplained P&L = Hypothetical P&L – Sensitivity P&L.
Backbone Adjustment

- Backbone adjustment is an advanced topic in sensitivity P&L.
- It can be best explained mathematically.
- Assume the value of an option is a function of the underlying price $S$ and implied volatility $\sigma$, i.e., $V = F(S, \sigma)$.
- If the implied volatility is a function of the ATM volatility and strike (sticky strike assumption), i.e., $\sigma = \sigma_A + f(K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A = DeltaP&L + VegaP&L$$

where $DeltaP&L = \frac{\partial F}{\partial S} dS$ and $VegaP&L = \frac{\partial F}{\partial \sigma_A} d\sigma_A$. 
Reference:
https://finpricing.com/lib/EqConvertible.html